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AN ASSESSMENT OF TWO-PHASE PRESSURE DROP CORRELATIONS FOR STEAM–WATER SYSTEMS

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Abstract—Eighteen two-phase friction pressure drop models and correlations were tested against about 2220 experimental steam–water pressure drop measurements under adiabatic conditions and about 1230 in diabatic flow conditions. The data represented several geometries and had the following property ranges:

Pressure	1.7–10.3 MN/m^2 (250–1500 psia);
Mass velocity	270–4340 kg/m^{2}sec (0.2–3.2 Mlb/ft^2 hr);
Quality	Subcooled to 100%;
Equivalent diameters	2.3–33.0 mm (0.09–1.3 in.).

The four models and correlations which were found to have the best performance were the Baroczy correlation, the Thom correlation and the homogeneous model two-phase friction multipliers,

$$\phi_{fo}^2 = \left[1 + x\left(\frac{v_{fg}}{v_f}\right)\right]$$

and

$$\phi_{fo}^2 = \left[1 + x\left(\frac{v_{fg}}{v_f}\right)\right]\left[1 + x\left(\frac{\mu_g}{\mu_f} - 1\right)\right]^{0.25}.$$

The correlations were also evaluated with the data being sub-divided into sets which were based on properties and flow conditions.

1. INTRODUCTION

A number of two-phase friction pressure drop correlations in the literature have been examined for their applicability to Boiling Water Reactor conditions. They were tested against about 2220 experimental steam–water pressure drop measurements under adiabatic conditions and about 1230 in diabatic flow conditions.

The experimental data had the following ranges:

Pressure: 1.7–10.3 MN/m^2;
Mass velocity: 270–4340 kg/m^2 sec;
Steam quality: subcooled to 100%;
Geometric configuration: tube, annulus, rectangular channel and rod array;
Equivalent diameter: 2.3–33.0 mm.

The correlations which were found to have the best overall performance were (a) homogeneous theory, (b) Thom (1964), and (c) Baroczy (1968). This study was made in partial fulfilment of a Master's Degree and the Thesis (Idsinga 1975) should be referred to for further details.

2. CORRELATIONS EXAMINED

Table 1 summarizes the correlations evaluated and also shows that a variety of friction factor and void fraction calculations were used in the development and application of correlations. The use of different void fraction and friction factor models can obviously affect the pressure drop predictions made by the correlations.

Table 1. A summary of two-phase correlations

Correlation of model	I.D. No.	Ref.	Method of application	Supporting data	Models used in development and/or application	
					Friction factor	void fraction
Homogeneous	1	—	$\phi_{fo}^2 = \left[1 + x\left(\frac{v_{fg}}{v_f}\right)\right]$	—	$f = 0.079/Re^{0.25}$	Homogeneous
Homogeneous	2	29	$\phi_{fo}^2 = \left[1 + x\left(\frac{v_{fg}}{v_f}\right)\right]\left[1 + x\left(\frac{\mu_f}{\mu_g} - 1\right)\right]^{-0.25}$	—	$f = 0.079/Re^{0.25}$	Homogeneous
Homogeneous	3	30	$\phi_{fo}^2 = \left[1 + x\left(\frac{v_{fg}}{v_f}\right)\right]\left[1 + x\left(\frac{\mu_g}{\mu_f} - 1\right)\right]^{+0.25}$	—	$f = 0.079/Re^{0.25}$	Homogeneous
Homogeneous	4	31	$\phi_{fo}^2 = \left[1 + x\left(\frac{v_{fg}}{v_f}\right)\right]\left[\frac{xv_g\left(\frac{\mu_g}{\mu_f}\right) + (1-x)v_f}{xv_g + (1-x)v_f}\right]^{+0.25}$	—	$f = 0.079/Re^{0.25}$	Homogeneous
Armand	5	(1959)	equation [4.9]	Air–water 0.1 MN/m^2	$f = 0.070/Re^{0.25}$	Same
Armand–Treschev	6	(1959)	equation [4.31]	Steam–water 1.0–18.6 MN/m^2	?	Same
Lockhart–Martinelli	7	(1969)	figure 4.1	Air–various liquids 0.1–0.3 MN/m^2	$f = 0.046/Re^{0.2}$	Same
Martinelli–Nelson	8	(1968)	figure 4.2	Steam–water 0.1–20.7 MN/m^2	$f = 0.079/Re^{0.25}$	Same
Bankoff	9	(1960)	equation [4.56]	Steam–water 6.9 MN/m^2	?	Same
Martinelli–Nelson–Jones	10	(1961)	equation [4.45]	?	Rough table	?
Levy momentum exchange	11	(1959)	equation [4.44]	Steam–water 0.4–9.8 MN/m^2	?	Same
Sze-Foo Chien & Ibele	12	(1962)	equation [4.57]	Air–water 0.1 MN/m^2	?	?
Thom	13	(1964)	table 4.5	Steam–water 0.1–20.7 MN/m^2	Rough tube	Same
Baroczy	14	(1968)	figure 4.5	Steam 1–13.8 MN/m^2	$f = 0.046/Re^{0.2}$	?
Becker	15	(1962)	equation [4.61]	Steam–water 0.7–4.1 MN/m^2	Rough tube	Martinelli–Nelson
Borishansky	16	(1973)	equation [4.63]	Steam–water 0.1–3.4 MN/m^2	$f = 0.046/Re^{0.2}$	?
Chisholm	17	(1973)	equation [4.66]	Other correlations	?	?
Lombardi & Peddrochi	18	(1972)	equation [4.67]	Steam–water 1.4–10.3 Mn/m^2	?	Homogeneous

The fluids used to develop the correlation are indicated in Column 5.

The expression for the friction factor used with the correlation is indicated in Column 6. In some cases, the appropriate expression was deduced from its use in the derivation of the correlation as in examples.

The homogeneous theory multiplier is defined as $\phi_{fo}^2 = (f_{TP}/f)(1 + x(v_{fg}/v_f))$. The two phase friction factor is calculated via the Reynolds Number which in turn uses a "two phase" viscosity. The viscosity is taken for correlations 1–4 as follows:

I.D. No. (1) $\bar{\mu} = \mu_f$

I.D. No. (2) $1/\bar{\mu} = x/\mu_g + (1-x)/\mu_f$

I.D. No. (3) $\bar{\mu} = x\mu_g + (1-x)\mu_f$

I.D. No. (4) $\bar{\mu} = (xv_g\mu_g + (1-x)v_f\mu_f)/\bar{v}$.

Refer to Idsinga (1975) for details of equations.

3. DATA USED

The data used in this evaluation are identified in table 2. These data represent measured total pressure drop results rather than only the reported friction related component of measured data. Measured total pressure drop data were chosen so that friction-related components could be obtained from all data using consistent void correlations and friction factors. This approach also permits calculation of the uncertainty in the resulting two-phase multiplier based on known or estimated experimental uncertainties.

The adiabatic and adiabatic data cover the pressure range 1.7–10.3 MN/m^2, the entire quality range and a large spectrum of mass velocities and configurations. These data specify the measured pressure drop or gradient, the flow conditions and geometry. Although investigators

Table 2. Data used in this study

Data Set (Note 1)	Ref.	Points	Configuration	Flow direction	De (mm)	Pressure range (MN/m^2)	Mass velocity range (kg/m^2sec)	Quality range	Mean data uncertainty	RMS data uncertainty
A-1	CISE (1961)	54	Rd. tube	Up	5.2	6.8–6.9	1085–3933	0.05–0.63	0.059	0.060
A-2	CISE (1961)	172	Rd. tube	Up	5.2	4.0–8.3	949–3933	0.01–0.71	0.063	0.068
A-3	CISE (1961)	49	Rd. tube	Up	5.0	6.8–6.9	949–4476	0.01–0.64	0.063	0.065
A-4	CISE (1961)	58	Rd. tube	Up	5.2	6.8–7.0	949–4069	0.02–0.73	0.059	0.060
A-5	CISE (1961)	74	Rd. tube	Up	6.3	6.8–7.0	949–4069	0.03–0.85	0.061	0.063
A-6	CISE (1961)	57	Rd. tube	Up	8.2	6.8–6.9	949–4069	0.15–0.65	0.059	0.059
A-7	CISE (1961)	27	Rd. tube	Up	10.1	6.8–7.0	1085–3255	0.03–0.75	0.069	0.075
A-8	CISE (1961)	61	Annulus	Up	5.0	6.8–7.2	1085–3526	0.04–0.76	0.060	0.061
A-9	CISE (1961)	68	Annulus	Up	7.0	6.8–7.1	1085–4611	0.01–0.72	0.067	0.075
A-10	CISE (1961)	151	Annulus	Up	3.2	6.8–7.1	949–3933	0.00–0.74	0.062	0.066
A-11	Gaspari *et al.* (1964)	51	Rd. pipe	Up	8.1	7.0	1085–3933	0.03–0.90	0.060	0.061
A-12	Gaspari *et al.* (1964)	72	Rd. pipe	Up	4.9	4.9–9.0	1085–3933	0.01–0.60	0.060	0.061
A-13	Gaspari *et al.* (1964)	360	Rd. pipe	Up	9.2	5.0–7.1	407–3933	0.02–1.0	0.066	0.072
A-14	Gaspari *et al.* (1964)	42	Rd. pipe	Up	15.2	5.0–5.1	949–1492	0.25–0.98	0.057	0.057
A-15	Gaspari *et al.* (1964)	268	Rd. pipe	Up	15.2	1.9–9.0	407–2034	0.02–0.98	0.122	0.444
A-16	Gaspari *et al.* (1964)	155	Rd. pipe	Up	5.1	6.9–7.1	949–3933	0.02–0.96	0.050	0.061
A-17	Gaspari *et al.* (1964)	66	Rd. pipe	Up	5.0	6.8–7.1	1085–3933	0.03–0.81	0.059	0.059
A-18	Gaspari *et al.* (1964)	13	Rd. pipe	Up	5.0	7.0–7.1	1085–1627	0.07–0.87	0.058	0.058
A-19	Gaspari *et al.* (1964)	26	Rd. pipe	Up	5.0	6.9–7.0	1492–3933	0.02–0.87	0.056	0.056
A-20	Gaspari *et al.* (1964)	37	Annulus	Up	2.5	5.0–8.1	678–3933	0.01–0.51	0.053	0.053
A-21	Era *et al.* (1966)	23	Annulus	Up	4.9	7.0–7.2	678–3119	0.01–0.52	0.081	0.094
A-22	Era *et al.* (1966)	22	Annulus	Up	4.9	7.0–7.2	1085–3933	0.00–0.53	0.151	0.221
A-23	Janssen & Kervinen (1964)	43	Rect. channel	Up	19.7	4.1–9.7	678–2848	0.02–0.99	0.166	0.225
A-24	Janssen & Kervinen (1964)	26	Rect. channel	Down	19.7	4.1–6.9	271–2848	0.02–0.79	0.131	0.149
A-25	Janssen & Kervinen (1964)	62	Rect. channel	Horiz.	19.7	4.1–9.8	271–2848	0.02–0.77	0.061	0.061
A-26	Janssen & Kervinen (1964)	23	Rect. channel	Up	11.1	4.1–9.7	678–2848	0.05–0.92	0.064	0.064
A-27	Janssen & Kervinen (1964)	18	Rect. channel	Horiz.	11.1	4.1–7.0	678–2848	0.05–0.90	0.062	0.062
A-28	Janssen & Kervinen (1964)	36	Rd. pipe	Up	24.3	4.1–9.6	271–1492	0.09–0.90	0.099	0.121
A-29	Janssen & Kervinen (1964)	44	Rd. pipe	Horiz.	24.3	4.1–9.6	271–1492	0.09–0.90	0.062	0.062
A-30	Janssen & Kervinen (1964)	14	Rd. pipe	Horiz.	32.3	6.9	271–813	0.09–0.90	0.062	0.062
A-31	Janssen & Kervinen (1964)	14	Rd. pipe	Horiz.	18.8	6.9	1085–2306	0.09–0.90	0.062	0.062
A-32	Janssen & Kervinen (1964)	37	Rd. pipe	Down	18.8	4.1–9.6	271–1492	0.09–0.90	0.045	0.104
A-33	Janssen & Kervinen (1971)	6	Rd. pipe	Up	17.3	6.8–7.1	1356–2170	0.05–0.25	0.150	0.183
D-1	Gaspari *et al.* (1964)	15	Rd. pipe	Up	5.0	6.8–7.1	949–1628	0.17–0.71	0.133	0.148
D-2	Gaspari *et al.* (1964)	121	Rd. pipe	Up	5.1	5.0–9.0	949–3933	0.31–0.96	0.073	0.077
D-3	Gaspari *et al.* (1964)	70	Rd. pipe	Up	5.1	6.9–7.1	949–3797	0.06–0.98	0.064	0.065
D-4	Gaspari *et al.* (1964)	159	Rd. pipe	Up	5.0	5.0–9.0	1085–3933	0.14–0.99	0.075	0.077
D-5	Gaspari *et al.* (1964)	270	Rd. pipe	Up	5.0	6.9–7.2	1085–4069	0.13–0.91	0.068	0.070
D-6	Gaspari *et al.* (1964)	71	Rd. pipe	Up	5.0	6.8–7.2	1085–4069	0.12–0.86	0.080	0.082
D-7	CISE (1961)	309	Rd. pipe	Up	5.2	4.1–11.0	949–3933	0.06–0.83	0.088	0.090
D-8	CISE (1961)	143	Rd. pipe	Up	5.2	6.8–7.0	1085–3933	0.01–0.84	0.071	0.072
D-9	Macbeth (1972)	12	Annulus	Up	6.9	6.9	1356–3526	0.06–0.74	0.102	0.105
D-10	Janssen & Kervinen (1971)	5	Rd. pipe	Up	17.3	6.8–6.9	1356–2170	0.0–0.30	1.46	1.46
D-11	Lahey *et al.* (1970)	31	Array	Up	12.0	6.9	271–2984	0.81–0.45	0.748	1.15
D-12	Mendler *et al.* (1960)	25	Rect. channel	Up	8.5	8.3	407–678	0.06–0.65	1.42	1.42

A—Adiabatic, D—Diabatic.

did not consistently state the surface finish of their test sections, the bulk of the data appears to be from smooth test configurations. Most data sets provided adequate information to establish the uncertainty in the measured variables; these error intervals are given in table 3. Some data were presented without the uncertainty range; for these a median value of the error interval for other experiments (as given in table 3) was applied. A considerable body of data available in the literature in graphical form was not utilized because associated uncertainty information was not available. It can be noted that this selection process yielded a data bank dominated by CISE and GE, San Jose data.

4. METHOD OF CORRELATION ASSESSMENT

4.1 *Correlation comparison with data*

The error in applying the correlation to each experimental data point was found as defined by

$$\epsilon \equiv \frac{(\phi_{fo}^2)_{\text{correlation}} - (\phi_{fo}^2)_{\text{exp}}}{(\phi_{fo}^2)_{\text{exp}}} \qquad [1]$$

where

ϵ = error

$(\phi_{fo}^2)_{\text{correlation}}$ = two phase friction multiplier calculated from the correlation;

$(\phi_{fo}^2)_{\text{exp}}$ = two phase friction multiplier from the experimental data.

For groups of data, the mean error, *RMS* error, and standard deviation of the error from the mean were also calculated as:

$$\text{Mean error } \bar{\epsilon} = \sum_{i=1}^{i=N} \epsilon_i/N, \qquad [2]$$

$$\text{RMS error } \epsilon_{\text{RMS}} = \left[\sum_{i=1}^{i=N} \epsilon_i^2/N\right]^{1/2}, \qquad [3]$$

$$\text{Standard deviation} = \left[\sum_{i=1}^{i=N} (\epsilon_i - \bar{\epsilon})^2/N\right]^{1/2} = [\epsilon_{\text{RMS}}^2 - \bar{\epsilon}^2]^{1/2}. \qquad [4]$$

4.2 *Data reduction*

Equation [5] below was used to reduce the pressure drop to a friction pressure gradient.

$$\left(\frac{\mathrm{d}p}{\mathrm{d}z}F\right) = \left(\frac{\mathrm{d}p}{\mathrm{d}z}\right) + G^2\frac{\mathrm{d}}{\mathrm{d}z}\left[\frac{x^2 v_g}{\alpha} + \frac{(1-x)^2}{1-\alpha}v_f\right] + g\sin\theta(\rho_g\alpha + (1-\alpha)\rho_f) \qquad [5]$$

where

$\left(\frac{\mathrm{d}p}{\mathrm{d}z}F\right)$, pressure gradient due to friction;

$\frac{\mathrm{d}p}{\mathrm{d}z}$, total pressure gradient;

G, mass velocity;

z, distance along flow path;

v_g, specific volume of saturated vapor;

α, void fraction;

v_f, specific volume of saturated liquid;

θ, orientation from vertical of test section;

ρ_g, saturated vapor density;

ρ_f, saturated liquid density.

Table 3. Uncertainty intervals for measured variables

Property	Uncertainty intervals						Median
Static pressure (P)	0.07 MN/m^2	0.035 MN/m^2	0.07 MN/m^2	0.035 MN/m^2		0.10 MN/m^2	0.07 MN/m^2
Pressure drop (ΔP)	2.5%	2.5%	270 N/m^2	1.2%	69 N/m^2	2.5%	2.5%
Mass flow rate (W)	1%	0.6%	2%	2%	1%	1%	1%
Diameters (D)	1%	1%	1%	1%			1%
Power to boiler (Φ_1)	3%	2%	1%	3%		2%	2%
Power to test section (Φ_2)	2%	1%	1%		2%	1%	1%
Inlet temperatures (T_{in})	7.2°C	3.6°C	9.0°C	7.2°C	3.6°C	7.2°C	7.2°C
Notes		2	1, 7	4		3, 5, 6	
Reference	CISE (1961) Berkowitz (1960)	Gaspari *et al.* (1964) Adorni *et al.* (1962)	Janssen & Kervinen (1971)	Janssen & Kervinen (1964)	Mendler *et al.* (1960)	Era *et al.* (1966)	Used on data of Lahey *et al.* (1970) Mendler *et al.* (1960) Macbeth (1972)

Notes to table:

1. The boiler is a heated length before test section pressure taps and is part of the same circuitry as the test section.
2. The boiler power uncertainty varied depending on measuring equipment attached between 1 and 2.2 percent.
3. The boiler power uncertainty calculated knowing a quality uncertainty of 0.02 at $x = 0$.
4. The pressure drop accuracy reported to be 0.3% of full scale of three manometers with liquids of different densities. The uncertainty is estimated based on manometer reading at $\frac{1}{4}$ length.
5. The pressure drop error is based on 1% of full scale for 2000 mm mercury manometer.
6. The Era *et al.* (1966) pressure drop uncertainty can be much higher than 2.5%.
7. The pressure drop uncertainty based on accuracy of static pressure profile accuracy of 138 N/m^2 over test section.

In the case of adiabatic data, the equation was used as given. The resulting relative magnitude of the friction pressure gradient relative to the total measured gradient varied principally with quality and mass velocity increasing as both these quantities increased. In the 0 to 0.1 quality range for $D_e = 12.7$ mm and $p = 1.7$ MN/m^2 the friction pressure gradient percentage of the total measured gradient varied approximately with mass velocity as follows; 1–11% for mass velocity of 400 kg/m^{2}sec; 14–62% for 1350 kg/m^{2}sec; 45–89% for 2700 kg/m^{2}sec and 69–96% for 4050 kg/m^{2}sec.

For diabatic data[5] was integrated in ten steps over the quality difference. In this case an average two-phase friction factor multiplier defined by[6] was calculated.

$$\frac{1}{x_{\text{out}} - x_{\text{in}}} \int_{x_{\text{in}}}^{x_{\text{out}}} \phi_{fo}^2 \, dx = \frac{\int_0^z \left(\frac{dp}{dz}F\right) dz}{\int_0^z \left(\frac{dp}{dz}F\right)_{fo} dz} \qquad [6]$$

where the two-phase friction multiplier is that defined in terms of the entire flow as liquid, i.e.

$$\phi_{fo}^2 \equiv \frac{\left(\frac{dp}{dz}F\right)}{\left(\frac{dp}{dz}F\right)_{fo}}. \qquad [7]$$

For diabatic data the relative magnitude of the pressure drop components varied with test section length and imposed quality changes. It is difficult to give general bounds on their relative magnitudes although acceleration components were generally at least comparable to friction components for the quality range 0–0.1.

The adiabatic raw data were reduced by several methods. Three methods of calculating the void fraction were used, the Thom correlation, the homogeneous model and the Martinelli–Nelson correlation. The Thom correlation was selected as the primary void fraction correlation because of its extensive steam–water data base. The Martinelli–Nelson void fraction correlation and the slip ratio of unity (homogeneous model) were used to determine the effects of the different void fraction models on the results. The homogenous void fraction model was used to provide data reduced by the same method as that on which the CISE correlation was based. It also provides for a completely homogeneous computation of data-based multipliers for comparison with the homogeneous model friction multipliers considered.

The effect of using various friction factors (f) in the data reduction process was also examined. The adiabatic data were reduced using both approximations involving the Reynolds Number (Re), namely:

$$f = \frac{0.046}{Re^{0.2}} \quad \text{and} \quad f = \frac{0.079}{Re^{0.25}} \qquad [8]$$

and the smooth tube friction factor given by

$$\frac{1}{\sqrt{f}} = 4.0 \log_{10}[Re\sqrt{f}] - 0.4. \qquad [9]$$

In evaluating the liquid-only friction pressure drop in [7], liquid-only Reynolds numbers from 20,000 to 600,000 were encountered yielding smooth tube friction factors of 0.003–0.007 per [9].

The diabatic data were reduced to an average multiplier using only the smooth tube friction factor and the Thom void fraction correlations.

Many of the diabatic data were for flows having subcooled inlet conditions. The location of the point of zero quality was determined using equilibrium thermodynamics. In the region from the inlet to this point, the flow was treated as a single-phase flow with a friction factor of 0.0075 being used for the *GE* rod bundle data (Lahey 1970), and 0.005 for other ducts.

4.3 *Error in experimental data*

The likely errors in the sets of experimental data were also examined because of their applicability to the comparison of correlations. For example, it would clearly be unreasonable to say that one correlation with a RMS error of (say) 20% was superior to one with 21% if the experimental RMS error were 30%. The evaluation of the likely experimental error was based on the Kline & McClintock (1953) procedure and is fully described in Idsinga (1975). It was found that the major components of uncertainty in the multiplier for adiabatic data were uncertainties in the measured mass velocity and pressure drop. The error range in diabatic data was found to be strongly influenced by the inlet subcooling or quality and the change in quality through the test section.

5. RESULTS

Each set of reduced adiabatic data was evaluated twice, once as sets based on the source of data and secondly as groupings of like properties and flow conditions. The property/flow condition groupings combined data of similar pressure ranges, quality ranges and mass velocity ranges. Table 4 gives the property/flow condition ranges from which 41 data subsets were formed. For diabatic data, the correlation multipliers were determined and averaged over the quality range of the data point.

Table 4. The ranges of physical properties and flow conditions used to form data subsets for evaluation by properties

Pressure:	$P < 6.3\,\mathrm{MN/m^2}$
	$P > 6.3\,\mathrm{MN/m^2}$
Mass velocity:	$G < 1356\,\mathrm{kg/m^2sec}$
	$1356\,\mathrm{kg/m^2sec} \le G < 2712\,\mathrm{kg/m^2sec}$
	$G \ge 2712\,\mathrm{kg/m^2sec}$
Quality:	$0 \le x < 0.1$,
	$0.1 \le x < 0.2$,
	$0.2 \le x < 0.3$,
	$0.3 \le x < 0.4$,
	$0.4 \le x < 0.5$,
	$0.5 \le x < 0.7$,
	$0.7 \le x < 1.0$.

41 data subsets were formed.

5.1 *Comparison of correlation with all data*

Table 5 gives the overall evaluation of adiabatic data for data reduction using the Thom void fraction correlation and a specific form of the single phase friction factor. This table gives the mean, the root-mean-square and the standard deviation of the error, ϵ, for all of the adiabatic data. Data reduction using different friction factors and void fraction models and correlations has also been performed by Idsinga (1975). The correlations are identified by numbers indicated in table 1. The terms data error and correlation error appearing in these tables refer to the uncertainty in the friction multiplier based on data and the discrepancy between data and correlations, respectively. Table 6 gives the overall evaluation of diabatic data.

5.2 *Comparison of correlations with various data groupings*

Table 7 gives the evaluation of adiabatic data groups reduced using the Thom void fraction correlation and the smooth tube single-phase friction factor. These results indicate how the correlations studied behave in different ranges of pressure, mass velocity and quality. The data groups are those identified in table 4.

For each data group the correlations are listed in order of increasing RMS error, ϵ_{RMS}, and include all correlations having ϵ_{RMS} within 0.1 of the best performing correlation.

Table 5. Overall results for adiabatic data reduced using the Thom void fraction correlation and the single-phase friction factor, $f = 0.046/Re^{0.2}$

Data sets	Points	Data MN error	Data RMS error	Correlation	Correlation MN error	Correlation RMS error	Correlation S.D.
33	2238	0.074	0.167	1	−0.092	*0.282	0.267
				2	−0.260	0.346	0.228
				3	−0.175	*0.305	0.250
				4	−0.331	0.390	0.207
				5	1.133	2.065	1.727
				6	0.025	0.364	0.363
				7	1.456	1.715	0.906
				8	0.478	0.648	0.437
				9	−0.229	0.539	0.488
				10	0.787	0.929	0.493
				11	0.359	0.834	0.753
				12	2.803	3.407	1.937
				13	−0.096	*0.282	0.265
				14	−0.088	0.310	0.297
				15	0.835	1.005	0.558
				16	0.145	0.372	0.343
				17	0.005	0.405	0.405
				18	0.276	0.488	0.403

Table 6. Overall results for diabatic data reduced with the Thom void fraction correlation and single-phase smooth tube friction factor

Data sets	Points	Data MN error	Data RMS error	Correlation	Correlation MN error	Correlation RMS error	Correlation S.D.
12	1231	0.127	0.298	1	−0.056	0.428	0.425
				2	−0.252	*0.408	0.320
				3	−0.155	*0.396	0.364
				4	−0.319	0.436	0.297
				5	1.122	1.938	1.581
				6	0.078	0.559	0.554
				7	1.554	1.929	1.143
				8	0.528	0.840	0.653
				9	−0.310	0.456	0.335
				10	0.720	0.991	0.681
				11	0.473	1.046	0.933
				12	3.446	4.299	2.571
				13	−0.064	0.423	0.418
				14	−0.198	*0.373	0.316
				15	0.969	1.333	0.916
				16	0.196	0.559	0.523
				17	−0.102	0.485	0.474
				18	0.129	0.443	0.424

6. DISCUSSION

6.1 *Correlations compared to adiabatic data*

6.1.1 ***Comparison with entire adiabatic data collection.*** Table 8 tabulates those correlations which had RMS correlation errors within 0.1 of the minimum in value. These lowest RMS correlation errors range from 0.25 to 0.30 while the RMS data uncertainty is much less, ranging from about 0.08–0.17. It is noted that, in general, the same correlations and models comprise this group regardless of how the data are reduced. Within this group the four correlations consistently exhibiting minimum error were: homogeneous equation with the viscosity term in the friction factor based on all liquid flow, I.D. No. 1; homogeneous equation with viscosity (μ) equal to $\bar{\mu} = x\mu_g + (1-x)\mu_f$, I.D. No. 3; Thom (1964), I.D. No. 13; and Baroczy (1968), I.D. No. 14.

The Chisholm (1973) correlation exhibits improved characteristics when the homogeneous and Martinelli–Nelson (1948) void correlations are used in the reduction of data. The RMS error for the Chisholm correlation falls just outside the arbitrary selection limit for Table 8 when the

Table 7. Correlations for adiabatic data subsets based on pressure, mass velocity and quality

Pressure (MN/m²)	Mass velocity (kg/m²sec)	Quality	Data points	ϵ_{RMS}, Correlation RMS error 0.1 0.2 0.3 0.4 0.75
1.7–6.2	0–1356	0–0.1	20	Nos. 12
		0.1–0.2	42	Nos. 16,14,13
		0.2–0.3	29	Nos. 14,11,16,13,1,3
		0.3–0.4	34	Nos. 14,11,13,16,1
		0.4–0.5	28	Nos. 14,16,11,13,18,8
		0.5–0.7	53	Nos. 18,16,8,17,6,13
		0.7–1.0	48	Nos.8,7,18
	1356–2712	0–0.1	30	Nos. 16,14,17,13,6,1,5,3,9
		0.1–0.2	37	Nos. 11,2,17,3,13,1,6
		0.2–0.3	28	Nos. 11,6,2,3,4
		0.3–0.4	31	Nos. 13,14,11,1,18,17,16,3
		0.4–0.5	17	Nos. 18,13,1,17,16,14,6,9
		0.5–0.7	23	Nos. 18,13,1,16,17,6
		0.7–1.0	17	Nos. 1,13,18,16
	2712–4068	0–0.1	13	Nos. 18,16,9
		0.1–0.2	8	Nos. 13,1,18,3,14,11,5
		0.2–0.3	9	Nos. 3,1,18,13,14,17,2
		0.3–0.4	9	Nos. 3,1,18,17
		0.4–0.5	9	Nos. 3,6,17,18
		0.5–0.7	9	Nos. 3,17,18,2
6.2–10.3	0–1356	0–0.1	67	Nos. 5,9,11,1,3,13,2,6,16,4
		0.1–0.2	86	Nos. 6,16,11,9,1,13,3,14
		0.2–0.3	79	Nos. 11,6,16,1,13,9,3
		0.3–0.4	68	Nos. 16,11,14,1,13
		0.4–0.5	54	Nos. 16,14,11
		0.5–0.7	110	Nos. 16,6,8,17,14
		0.7–1.0	94	Nos. 8,16,17,1,13
	1356–2712	0–0.1	107	Nos. 9,11,6,1,5,3,13,14,17,2,16,4
		0.1–0.2	143	Nos. 6,11,9,1,13,14,3,17,16,5,2
		0.2–0.3	95	Nos. 14,11,17,1,13,9,6,16,3,18
		0.3–0.4	90	Nos. 14,17,1,13,3,9
		0.4–0.5	77	Nos. 17,1,13,14,6,3
		0.5–0.7	129	Nos. 1,13,17,3
		0.7–1.0	63	Nos. 1,13,17,16,3,14
	2712–4068	0–0.1	84	Nos. 16,9,11,5,1,3,6,13,17,2,14,18
		0.1–0.2	90	Nos. 9,1,13,3,11,18,5,14,2,17,6
		0.2–0.3	76	Nos. 5,3,17,2,9,13,1,14,4
		0.3–0.4	63	Nos. 9,14,4,2,17
		0.4–0.5	57	Nos. 17,2,4,9,3
		0.5–0.7	69	Nos. 17,2,4,3
		0.7–1.0	27	Nos. 3,17,2,4,18

other methods of reducing data are employed. In most cases, including the Chisholm work, the difference in results by using the different models for void fraction and single phase pressure drop in data reduction is at best equal to the uncertainty in the data.

The CISE, Lombardi & Peddrochi (1972), correlation RMS error decreases significantly when the homogeneous model is used to calculate the void fraction in reducing data. This coincides with the fact that the homogeneous model was used to develop that correlation. The CISE correlation may be strongly affected by the friction factor used. It is noted that no friction factor is used in applying this correlation and none was needed to develop it. In this study the friction factor is used to calculate a liquid-only friction pressure drop which is then divided into the pressure drop determined by the correlation to convert it to a friction multiplier for

Table 8. Two-phase pressure drop correlations and models having the least discrepancy with the adiabatic data collection

Data reduction method

Friction factor		$f = 0.046/Re^{0.2}$	$f = 0.079/Re^{0.25}$	Smooth tube	Smooth tube	Smooth tube
Void fraction		Thom	Thom	Thom	Martinelli Nelson	Homogeneous model
Ranking	1	Homogeneous No. 1	Homogeneous No. 1	Homogeneous No. 1	Thom No. 13	Baroczy No. 14
	2	Thom No. 13	Thom No. 13	Thom No. 13	Homogeneous No. 1	Thom No. 13
	3	Homogeneous No. 3	Baroczy No. 14	Homogeneous No. 3	Baroczy No. 14	Homogeneous No. 1
	4	Baroczy No. 14	Homogeneous No. 3	Baroczy No. 14	Homogeneous No. 3	Homogeneous No. 3
	5	Homogeneous No. 2	Borishansky No. 16	Homogeneous No. 2	Homogeneous No. 2	Chisholm No. 17
	6	Armand–Treschev No. 6	Armand–Treschev No. 6	Armand–Treschev No. 6	Chisholm No. 17	Homogeneous No. 2
	7	Borishansky No. 16	Homogeneous No. 2	Borishansky No. 16	Borishansky No. 16	Armand–Treschev No. 6
	8	—	—	—	Armand–Treschev No. 6	—

Correlations having ϵ_{RMS} within 0.1 of the minimum.

comparison with data. This study is not a wholly valid evaluation of the CISE correlation since no friction factor is required for calculations as in other correlations and models.

It is noted that the three correlations based on data at pressures near one atmosphere display the greatest difference with data.

6.1.2 *Comparison with groupings of adiabatic data.* These results have been presented in Table 7 from which the following characteristics can be observed. The RMS correlation errors for the low pressure data (1.7–6.2 MN/m^2) are less than for the high pressure data (6.2–10.3 MN/m^2). Within each pressure group, the RMS correlation errors decrease with increasing value of mass velocity. Within each mass velocity group, the RMS correlation errors tend to decrease with quality values to about 0.4–0.5 and then increase. The 1350–2700 kg/m^{2}sec data group at 1.7–6.2 MN/m^2 is an exception to this later trend. Finally it can be observed that a larger number of correlations tend to match the data within the 0.1 RMS selection criteria for the lower quality groups (0.0–0.3) than for the higher quality groups.

Scanning table 7, it is not immediately obvious that the previously determined superiority of correlations Nos 1, 3, 13 and 14 with respect to the overall data bank is confirmed. In this regard due note should be taken of the preponderance of data points in the higher pressure groupings where these correlations perform well. The good performance of the homogeneous correlations probably also derives in part from this preponderance of high pressure data. Figure 1 illustrates some CISE pressure gradient data[23] over the range of conditions investigated in this study. The near linearity of the data at high pressure, moderately high mass flow favors the performance of correlation No. 3 which is defined to match all liquid and all vapor end points. This linearity in pressure gradient with quality as well as the linearity to qualities of about 0.8–0.9 for other pressure, mass flow conditions is also responsible for the favorable performance of correlation No. 1 which has a trajectory with quality which matches the data well over the quality range 0–0.8.

However as figure 1 shows, significant deviations from linearity occur in the friction pressure gradient under many sets of conditions so that the optimum correlation for specific data set is generally not a homogeneous model as table 7 demonstrates. Therefore users with the need for a friction pressure drop correlation over a specific data range should consult table 7. Additional details of correlation performance by data group are available in Idsinga (1975). Recommendation for correlations covering combinations of data groups comprising BWR conditions are given in section 7.

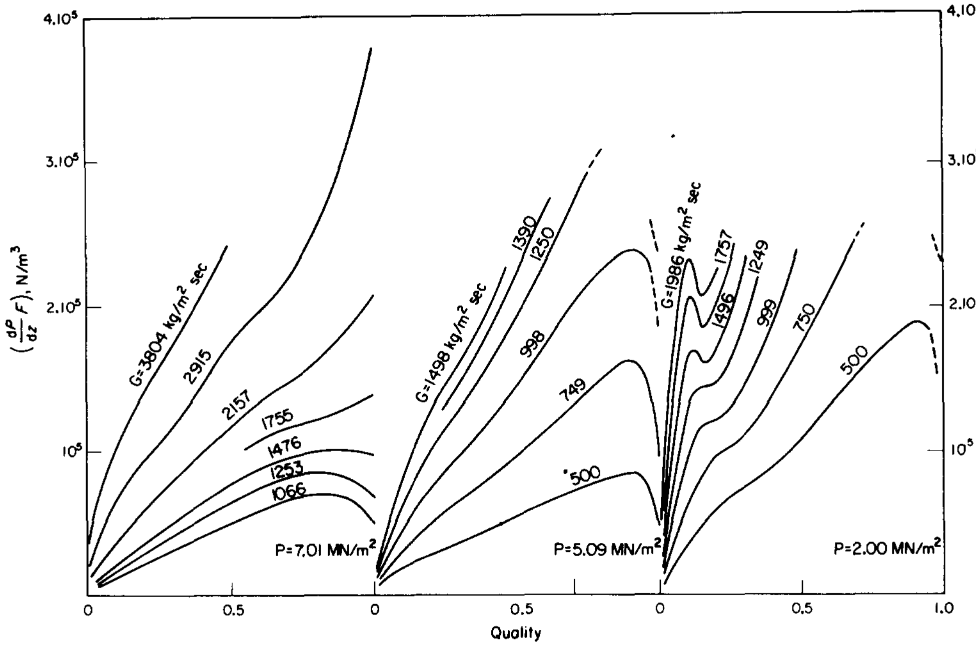

Figure 1. Measured two-phase friction pressure gradients from Gaspari *et al.* (1964).

6.2 *Correlations compared to diabatic data*

The correlations having the least discrepancy with the data are much the same as for the adiabatic data although there is some shifting of positions. There is greater uncertainty in the diabatic data, particularly when there is subcooling. This greater scatter is naturally reflected by the higher RMS discrepancies between correlation and data.

7. APPLICABILITY OF RESULTS TO BOILING WATER REACTOR ANALYSIS

Boiling Water Reactors operate within the limits of the data used in this study. Marinelli & Pastori (1972) report a comparison of predictions of a limited number of correlations under steady state BWR conditions. In previous sections a larger number of correlations have been compared against measured data for conditions representing transient as well as steady state BWR operation. The data subsets investigated that are pertinent to the normal operation of the BWR are those representing the following properties:

Pressure: 6.2–10.3 MN/m^2;
Mass velocity: 0–1350, 1350–2700 kg/m^{2}sec;
Quality: 0–0.1, 0.1–0.2.

The correlation which had the least RMS error overall for these conditions is the Armand–Treschev (1959) correlation.

In the event of a reactor transient such as the loss of coolant accident, the quality can be as high as (say) 0.6. Under these circumstances, conditions above expanded in quality to 0.6 are applicable. The Armand–Treschev correlation performed best up to a quality of 0.3. At these higher qualities the Baroczy correlation gave the best results.

A typical BWR 8 × 8 rod bundle has an equivalent diameter of 13.6 mm. The Thom and Baroczy correlations perform the best but the Armand–Treschev correlation also performed well in the sets having equivalent diameters near 13 mm. Since these geometry data sets included conditions of high velocities and qualities, the results are considered applicable to BWR conditions. Therefore, the Armand–Treschev correlation is recommended for BWR pressure drop analysis at qualities of less than 0.3 and the Baroczy correlation for higher qualities.

For applicability to pressurized water reactors a similar study should be conducted on steam–water pressure drop data at higher pressures than those examined here.

8. CONCLUSIONS

The performance of the eighteen two-phase friction pressure drop correlations evaluated in this study were expressed with regard to the total data bank, data sets representing specific ranges of pressure, mass velocity and quality and data sets representing BWR conditions. The RMS error between correlation prediction and data was selected as the criteria upon which to evaluate correlation performance.

1. Considering the total data bank, the four correlations exhibiting minimum error were (a) the homogeneous model with the two phase viscosity term based on all-liquid flow (No. 1), (b) the homogeneous model with the two phase viscosity equal to $\bar{\mu} = x\mu_g + (1-x)\mu_f$ (No. 3), (c) Thom (No. 13), (d) Baroczy (No. 14).
2. The best performing correlations for each data range can be determined by detailed reference to table 7.
3. For BWR friction pressure drop analysis, the Armand–Treschev correlation is recommended for qualities less than 0.3 and the Baroczy correlation for higher qualities to 0.6.

The selection of these correlations was investigated and found relatively independent of the friction factor and void fraction correlation used in reduction of the raw total pressure drop data.

REFERENCES

Adorni, N., Alessandri, A., Bertoletti, S., Gaspari, G. P., Lombardi, C., Peterlongi, G., Soldaini, G. & Zavattarelli, R. 1962 Description of a steam loop for heat-transfer experiment with steam-water mixtures (Part I) and of pressure drop measurements in adiabatic conditions (Part II). CISE-R-62.

Armand, A. A. 1959 The resistance during the movement of a two-phase system in horizontal pipes. *AERE-Trans.* 828.

Armand, A. A. & Treschev, G. G. 1959 Investigation of the resistance during the movement of steam-water mixtures in a heater boiler pipe at high pressures. *AERE-Lib/Trans.* 816.

Bankoff, S. G. 1960 A variable density single-fluid model for two-phase flow with particular reference to steam–water flow. *J. Heat Transfer* **82**, 265–272.

Baroczy, C. J. 1968 A systematic correlation for two-phase pressure drop. *Chem. Engng Prog. Symp. Ser.* No. 64.

Becker, K. M., Hernborg, G. & Bode, M. 1962 An experimental study of pressure gradients for flow of boiling water in vertical round ducts (Part 4). AE-86.

Berkowitz, L. *et al.* 1960 Results of wet steam cooling experiments: pressure drop, heat transfer and burnout measurements with round tubes. CISE-R-27.

Borishansky, V. M., Paleev, I. I., Agafonova, F. A., Andreevsky, A. A., Fokin, B. S., Lavrentiev, M. E., Malyus-Malitsky, K. P., Fromzel, V. N. & Danilova, G. P. 1973 Some problems of heat transfer and hydraulics in two-phase flows. *Int. J. Heat Mass Transfer* **16**, 1073–1085.

Chien, Sze-Foo & Ibele, N. 1962 Pressure drop and liquid film thickness of two-phase annular and annular-mist flows. ASME paper 62-WA-170.

Chisholm, D. 1973 Pressure gradients due to friction during the flow of evaporating two-phase mixtures in smooth tubes and channels. *Int. J. Heat Mass Transfer* **16**, 347–358.

Cicchitti, A. *et al.* 1960 Two-phase cooling experiments—pressure drop, heat transfer and burnout measurements. *Energia Nucl., Milano* **7**, 407–425.

Cise 1961 *A Research Program in Two-Phase Flow.*

Dukler, A. E., Wicks, M., III, & Cleveland, R. G. 1964 Pressure drop and hold-up in two-phase flow. Part A—A comparison of existing correlations. Part B—An approach through similarity analysis. *A.I.Ch.E. Jl.* **10**, 38–51.

ERA, A. & GASPARI, G. P. 1966 Heat transfer data in the liquid deficient region for steam–water mixtures at 70 kg/cm^2 flowing in tubular and annular conduits. CISE-R-184.

GASPARI, G. P. *et al.* 1964 Pressure drops in steam–water mixtures. CISE-R-83.

IDSINGA, W. 1975 An assessment of two-phase pressure drop correlations for steam–water systems. M.S. Thesis, Massachusetts Institute of Technology.

JANSSEN, E. & KERVINEN, J. A. 1964 Two phase pressure drop in straight pipes and channels: steam–water mixtures at 600 to 1400 psia. GEAP-4616.

JANNSEN, E. & KERVINEN, J. A. 1971 Developing two-phase flow in tubes and annuli. Part I, GEAP-10341.

JONES, A. B. 1961 Hydrodynamic stability of a boiling channel. DAPL-2170.

KLINE, S. J. & MCCLINTOCK, F. A. 1953 Describing uncertainties in single sample experiments. *Mech. Engng* **75**, 3.

LAHEY, R. T., JR., SHIRALKAR, B. S. & RADCLIFFE, D. W. 1970 Two phase flow and heat transfer in multirod geometries: subchannel and pressure drop measurements in nine-rod bundle for diabatic and adiabatic conditions. GEAP-13049.

LEVY, S. 1959 Steam slip—theoretical prediction from momentum model. ASME paper 59-HT-15.

LOCKHART, R. W. & MARTINELLI, R. C. 1949 Proposed correlation of data for isothermal two-phase, two component flow in pipes. *Chem. Engng Prog.* **45**, 39.

LOMBARDI, C. & PEDDROCHI, E. 1972 A pressure drop correlation in two-phase flow. *Energia Nucl., Milano* **19**, 91.

MCADAMS, W. H. *et al.* 1942 Vaporization inside horizontal tubes—II—benzene oil mixtures. *Trans. Am. Soc. Mech. Engrs* **64**, 193.

MACBETH, R. V. 1972 The effect of 'crud' deposits on the functional pressure drop in a boiling channel. AEEW-R 767.

MARINELLI, V. & PASTORI, L. 1972 Pressure drop calculations in BWR rod bundles. *Trans ANS* **15**, 412.

MARTINELLI, R. C. & NELSON, D. B. 1948 Prediction of pressure drop during forced-circulation boiling of water. *Trans. Am. Soc. Mech. Engrs* **70**, 695.

MENDLER, O. *et al.* 1960 Natural-circulation tests with water at 800 to 2000 psia under non-boiling, local boiling and bulk boiling conditions. ASME paper 60-HT-36.

SHER, N. C. & GREEN, S. J. 1959 Boiling pressure drop in thin rectangular tubes. *Chem. Engng Prog. Symp. Ser.* No. 23.

THOM, J. R. S. 1964 Prediction of pressure drop during forced circulation boiling of water. *Int. J. Heat Mass Transfer* **7**, 709–724.